Phase-slips and Coulomb blockade

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illustrated with devices

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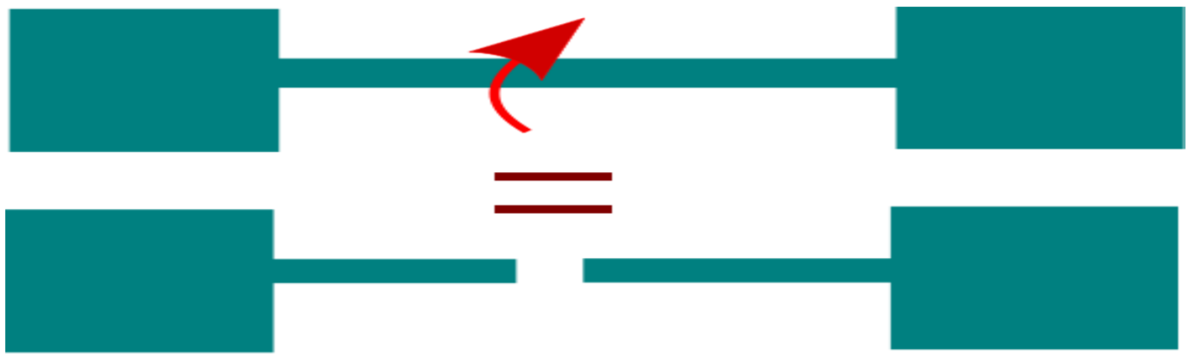
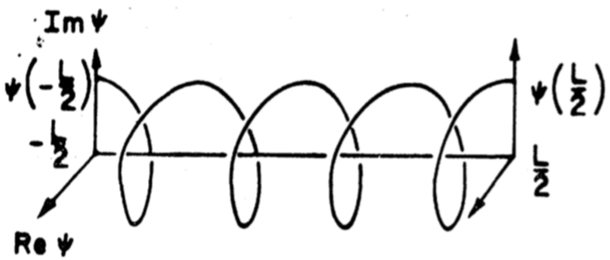
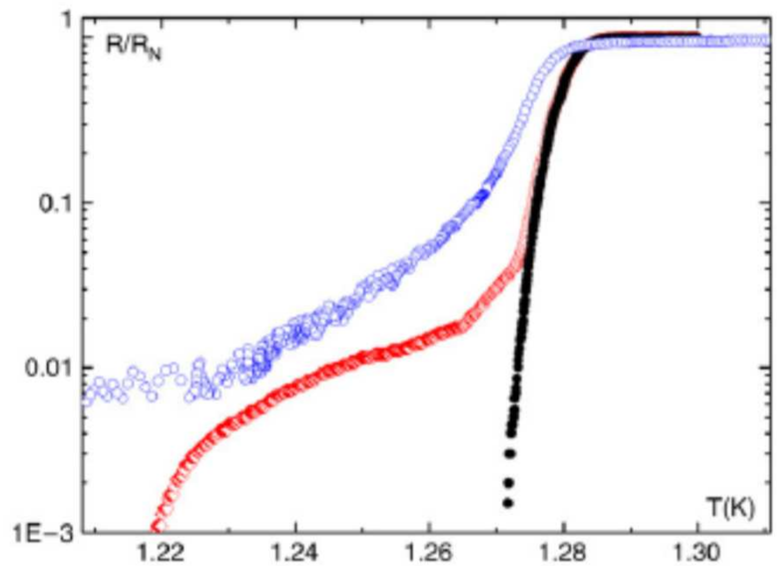
Hans Mooij, Kees Harmans, Ad Verbruggen, Tomoko Fuse



Introduction: phase-slips

- (Thin) superconducting wires near Tc
- Remain resistive – retain superconductivity
  - Sup. **Phase slips** some time, somewhere
- From 1967
  - M. Tinkham, V. Ambegaokar, W.A. Little, M. R. Beasly, B. I. Halperin, J. Clarke, J.E. Mersereau, I. O. Kulik, V.P. Galaiko, J. E. Mooij, T. M. Klapwijk, B. I. Ivlev, N. B. Kopnin, A. Baratoff …

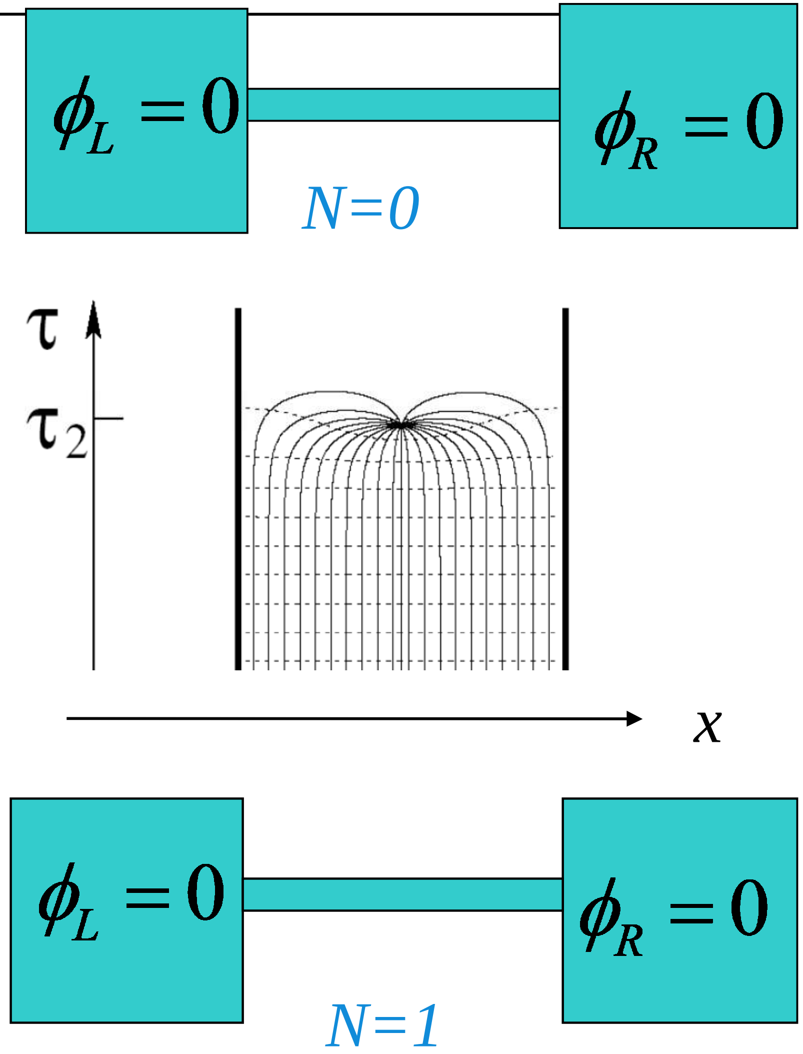




Topological meaning

- continuous profile of phase
- Phase defined upon 2π
- Stat. States – differ by winding number N
- Transitions between the states – phase slips
- Abrikosov vortex in ***x-t*** plane

- Solution of dynamical equations - real
- Solution of classical st. equations – classical fluctuations
- Saddle point of quantum action -Quantum fluctuation



Quest for quantum phase-slips

- Apply current, measure voltage, go to low T, see saturation of resistance
- 80’s: Giordano
- 00-10’s: Tinkham, Bezryadin, Arutyunov
- ?!?

- Some alternatives:
- Bezryadin 2009
- 2010 Guichard: MQT in Josephson chains

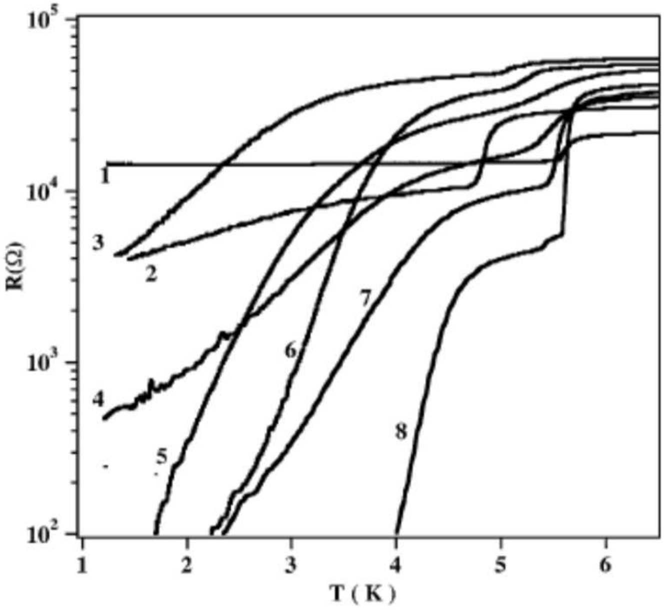
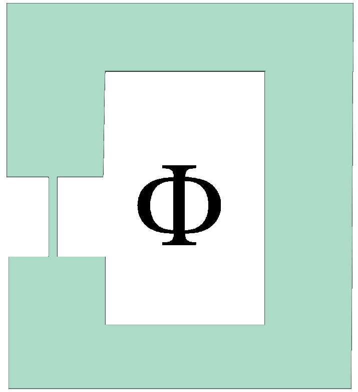
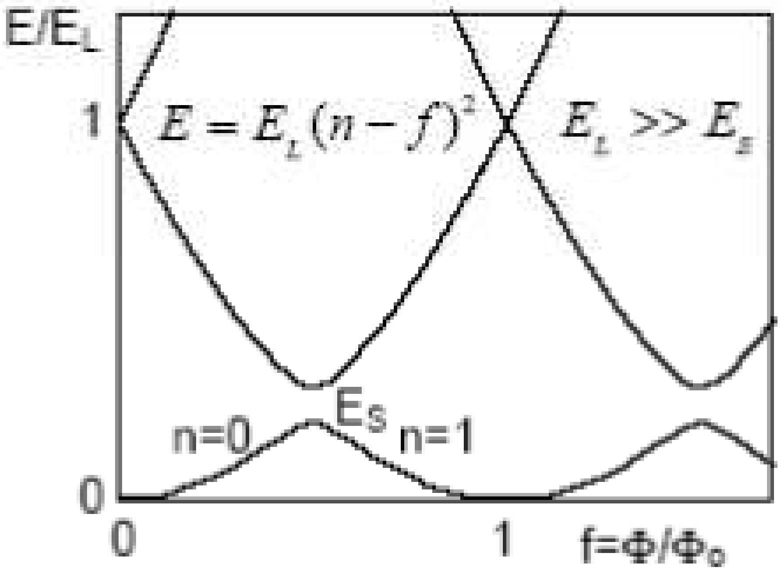


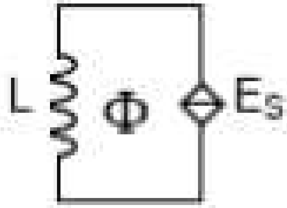
FIG. 1. Resistances as a function of temperature for eight different samples. The samples’ normal state resistances and lengths are 1: 14.8 kΩ, 135 nm; 2: 10.7 kΩ, 135 nm; 3: 47 kΩ, 745 nm; 4: 17.3 kΩ, 310 nm; 5: 32 kΩ, 730 nm; 6: 40 kΩ, 1050 nm; 7: 10 kΩ, 310 nm; 8: 4.5 kΩ, 165 nm.

Phase-slip qubit: alternative

- Mooij, Harmans 2005
  - Better be quantum-coherent







E_S – phase slip amplitude

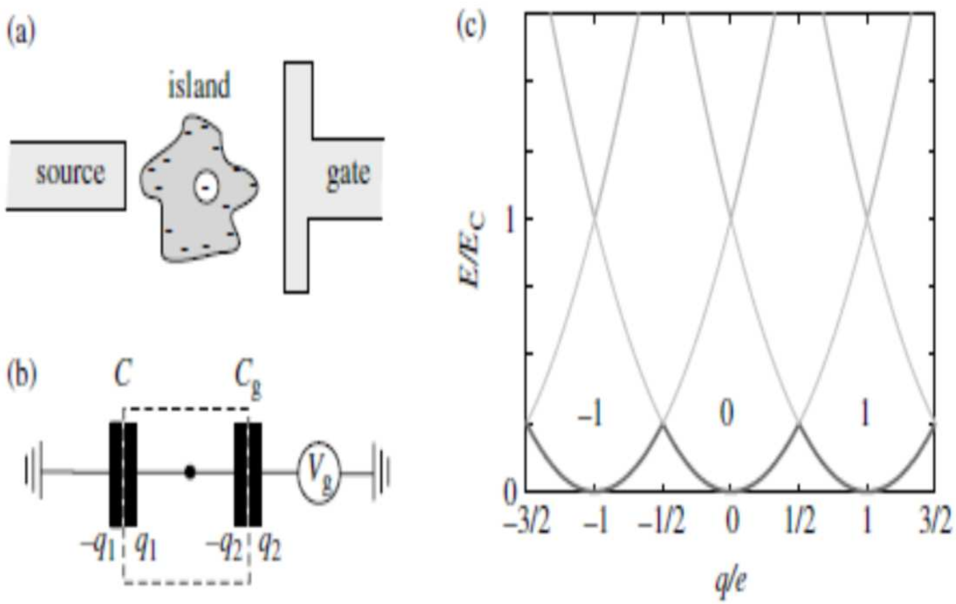
$$H_{QPS} = E_L(n-f)^2 - (\frac{E_S}{2}\sum_n |n+1\rangle\langle n| + h.c.)$$

Introduction Coulomb blockade

- Discrete charge N
  - (e or 2e)
- Charging energy

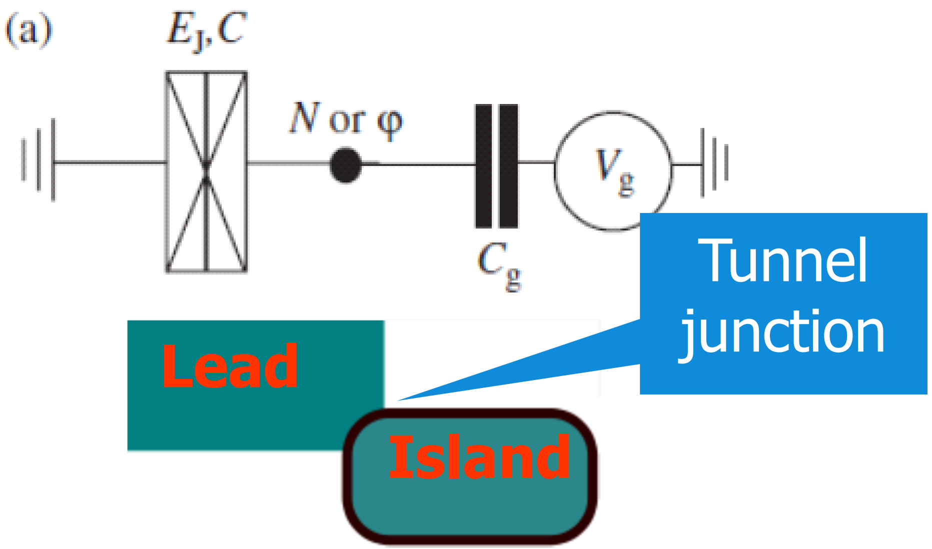
$$E_{\mathrm{el}} = E_{\mathrm{C}}\left(N - \frac{q}{e}\right)^2$$

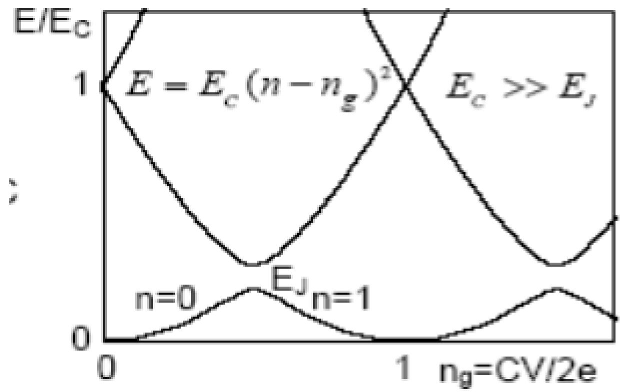
- Number of charges
  - tunable by gate voltage



Cooper Pair Box

- Energies
  - Josephson E_J
  - Charging E_C
- Limits
  - Discrete charges $\boxed{E_C \gg E_J}$
  - Exponentially small charge sensitivity $E_C \ll E_J$



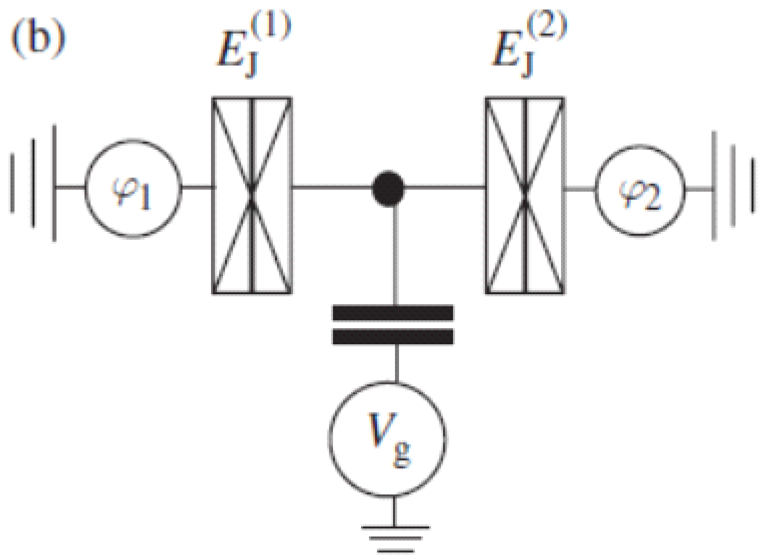


a. Cooper pair box

Cooper pair "transistor"

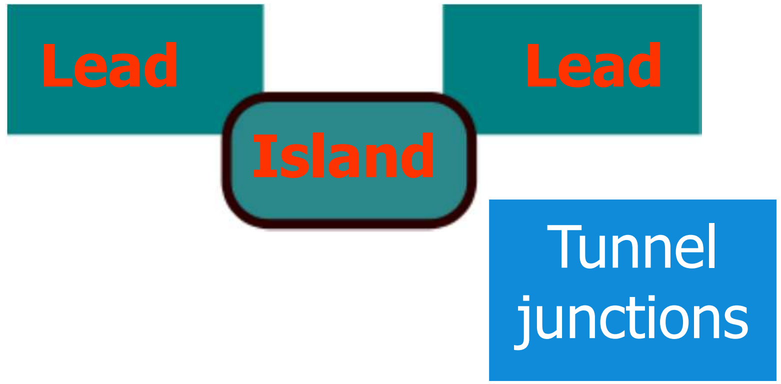
- Supercurrent runs through
- Modulated by gate voltage
- Hamiltonian is the same, amplitudes add

$$E_J = E_J^{(1)} \exp(i\phi_1) + E_J^{(2)} \exp(i\phi_2)$$



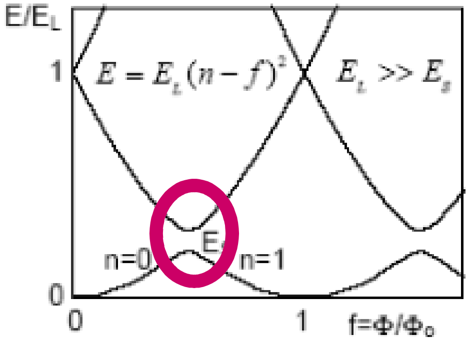
- Current suppression at

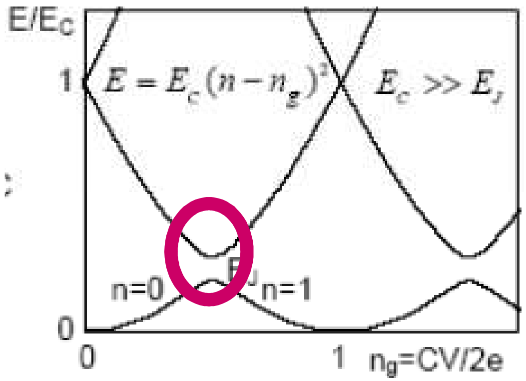
$$E_C \gg E_J \qquad I \sim \frac{E_J^2}{E_C}$$



- Non-exponential

Bringing together: through duality





a. Cooper pair box

- Cooper pair box,
  - (n=quantized charge)

$$H_{JJ} = E_C(n-n_g)^2 - (\frac{E_J}{2}\sum_n |n+1\rangle\langle n| + h.c.)$$

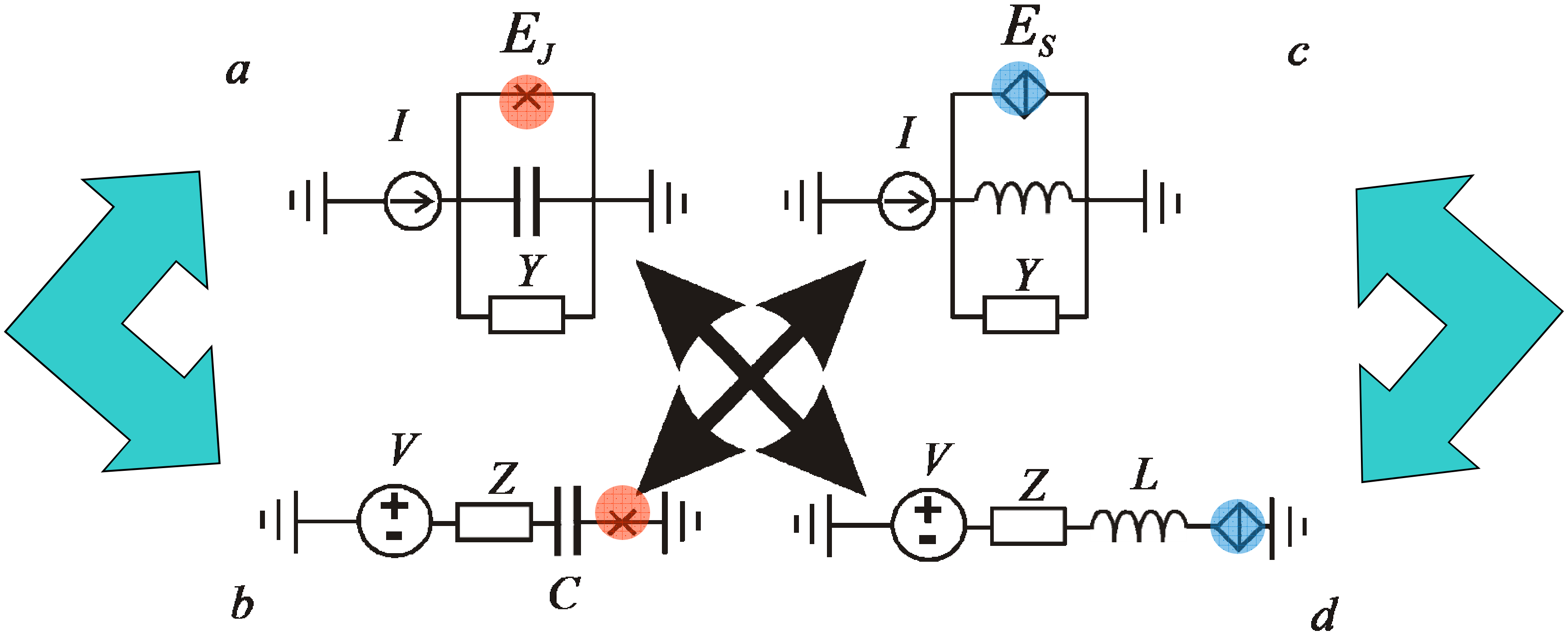
- QPS qubit
  - (n=winding number)

$$H_{QPS} = E_L(n-f)^2 - (\frac{E_S}{2}\sum_n |n+1\rangle\langle n| + h.c.)$$

- Not yet quite "dual"

Mooij, Nazarov 2006 $[\hat{q}, \hat{\phi}] = -i$

Duality: exact meaning $\hat{q} \to \hat{\phi}/2\pi, \hat{\phi} \to -2\pi\hat{q}$



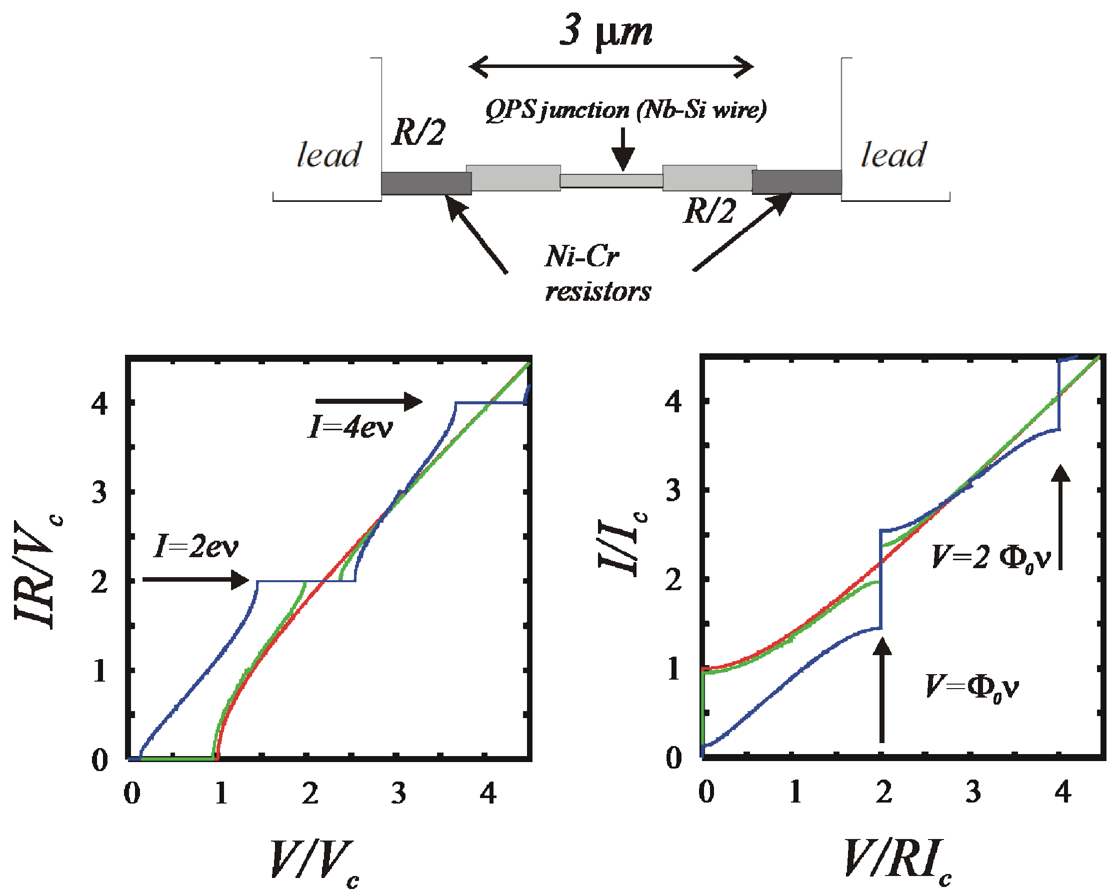
$E_S \cos\hat{q} \leftrightarrow E_J \cos\hat{\phi}$

$E_S \to E_J; \quad E_L \to E_C;$

$I \leftrightarrow R_q^{-1} V; Y(\omega) \leftrightarrow R_q^{-1} Z(\omega)$

Just flip V and I

- voltage (current) standard
- Supercurrent ⇔ Coulomb blockade



Macro and micro

- Phase-slip: instanton in complex 1+1 action

  $$\Delta(x,t), V(x,t)$$

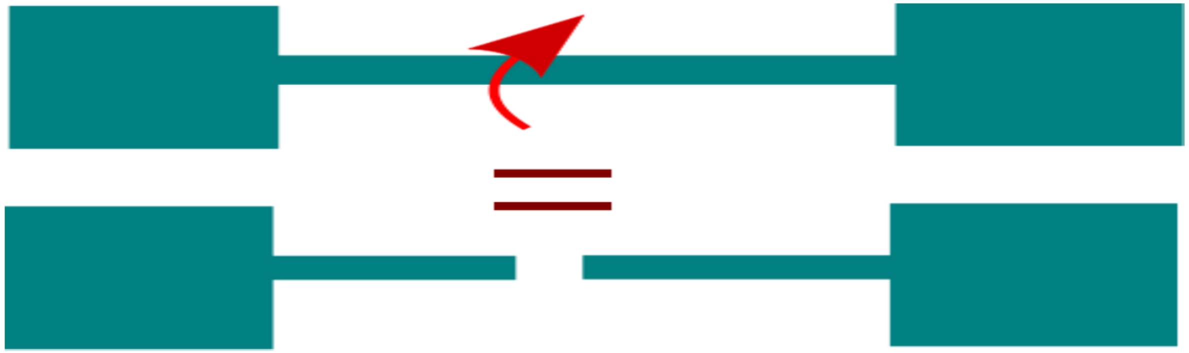
  - Sup. Quasiparticles energy
  - El. Excitations inside the wire: energy
  - No reliable theory

$$E_s \propto \exp\left\{-\frac{\alpha}{R G_Q}\right\}; \quad R = \frac{dR}{dx}\xi; \quad \alpha - \text{unknown}$$

- Reduction: to work at smaller energies
  - Phase drop or charge transferred
  - Wire = zero-dimensional
  - A single amplitude $\boldsymbol{E_s}$: phenomenological

Central point

- Phase-slip = Coulomb blockade=isolation
- Yet to be seen experimentally
- To facilitate this:
  - Devise devices
  - “investigate” those



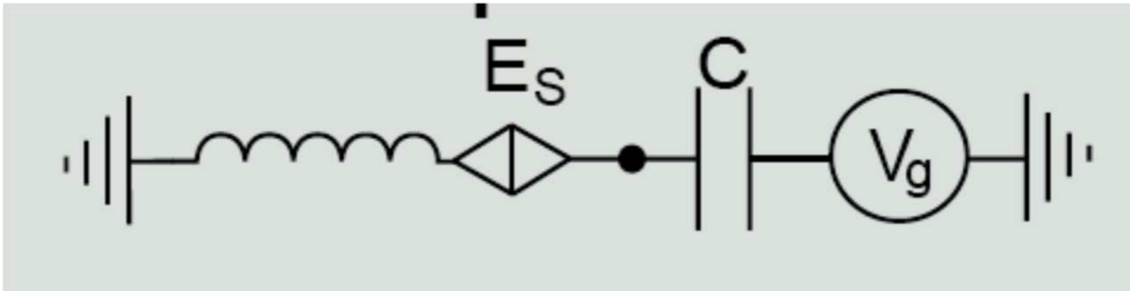
Phase-slip Cooper Pair Box

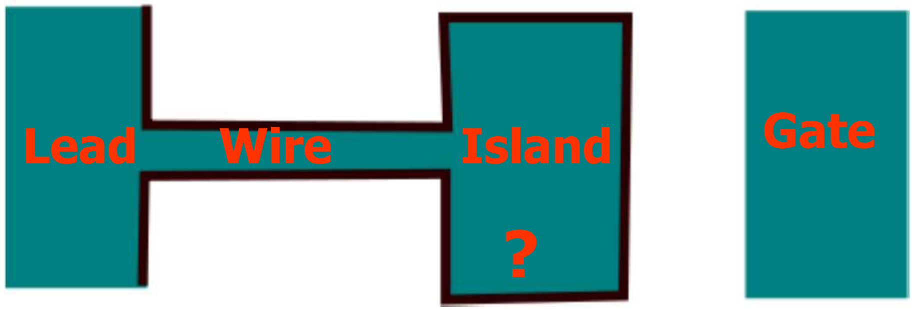
- Energies:
  - Inductive E_L
  - Charging E_C
  - PS E_s
- Hamiltonian
  - Continuous charge

$$E_S \cos(2\pi(q - q_0)) +$$

$$E_C q^2 / 2 +$$

$$E_L \phi^2 / 2$$

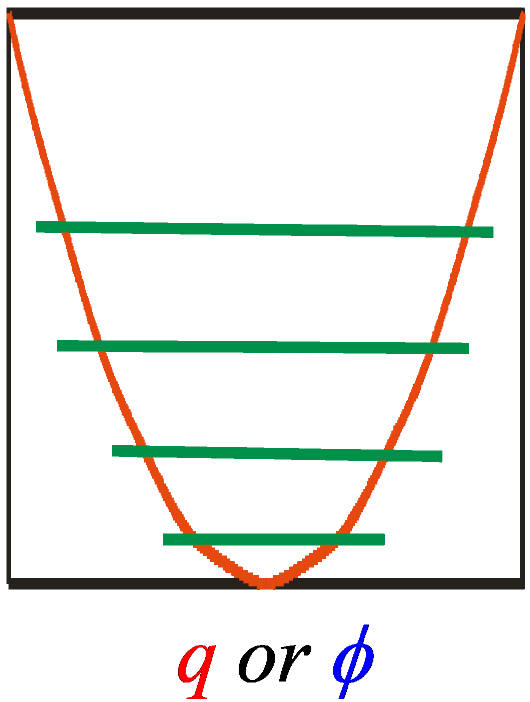




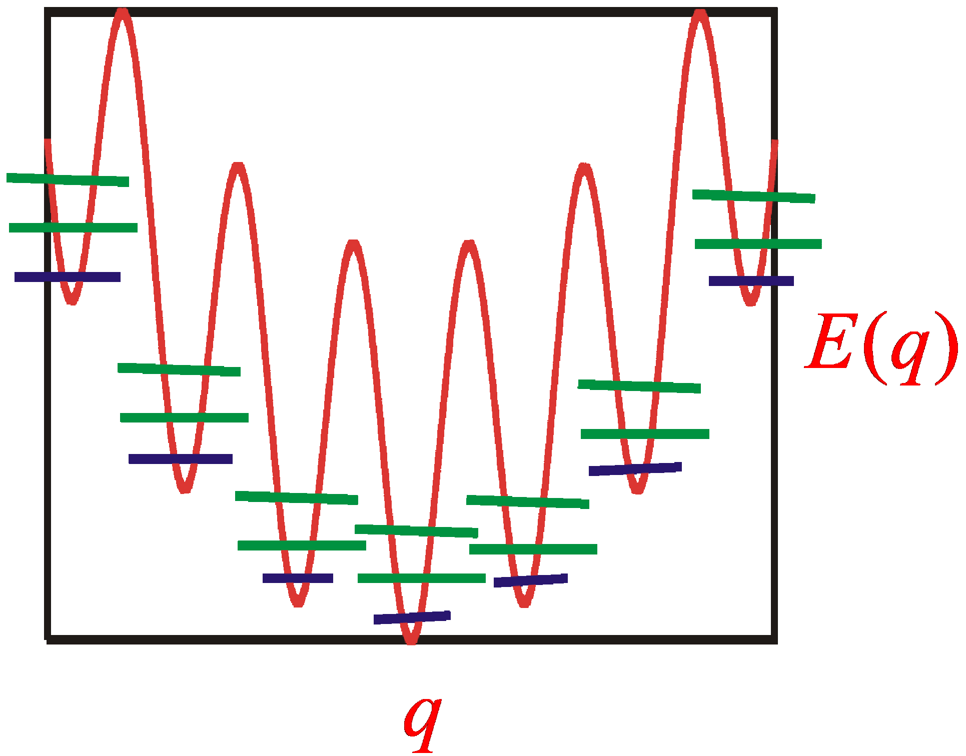
No Tunnel junctions

Limits

$E_s \ll E_L, E_C$

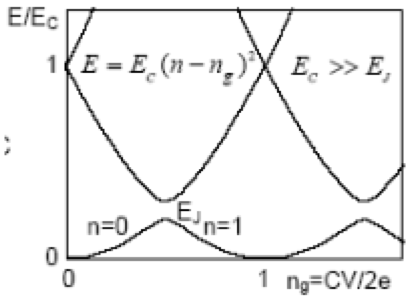


$E_s \gg E_L, E_C$



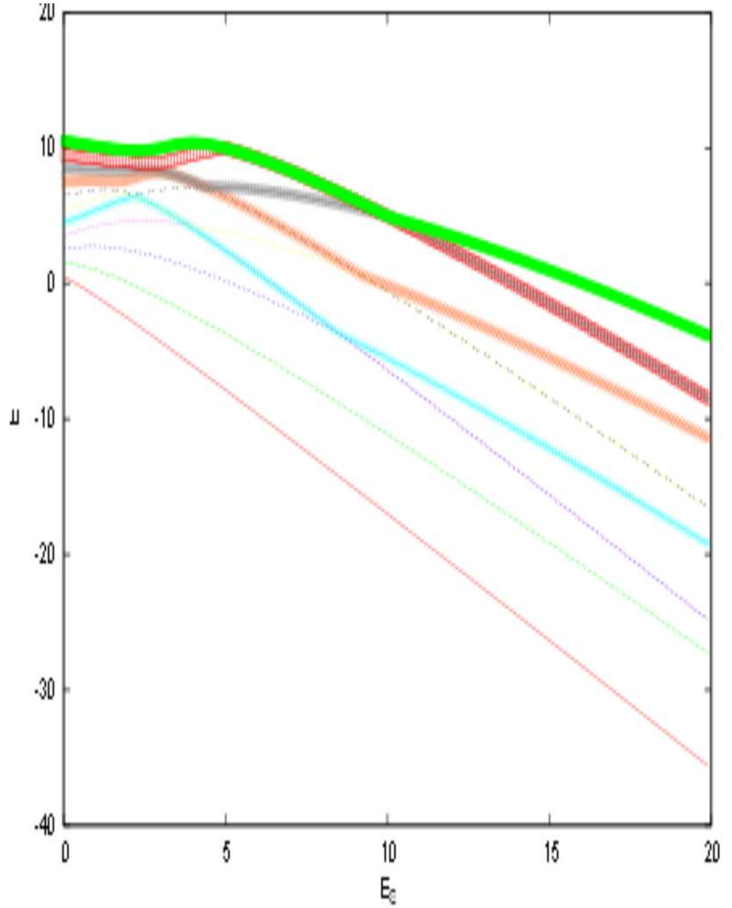
Peculiarities

- From oscillator
  - to oscillators
- Charge sensitivity
  - First-order $E_S \cos(2\pi q_0)$
  - Developed isolation
    - Exp-small gaps



a. Cooper pair box

$$\sqrt{E_S E_L} \gg E_C$$



Phase-slip CP transistor

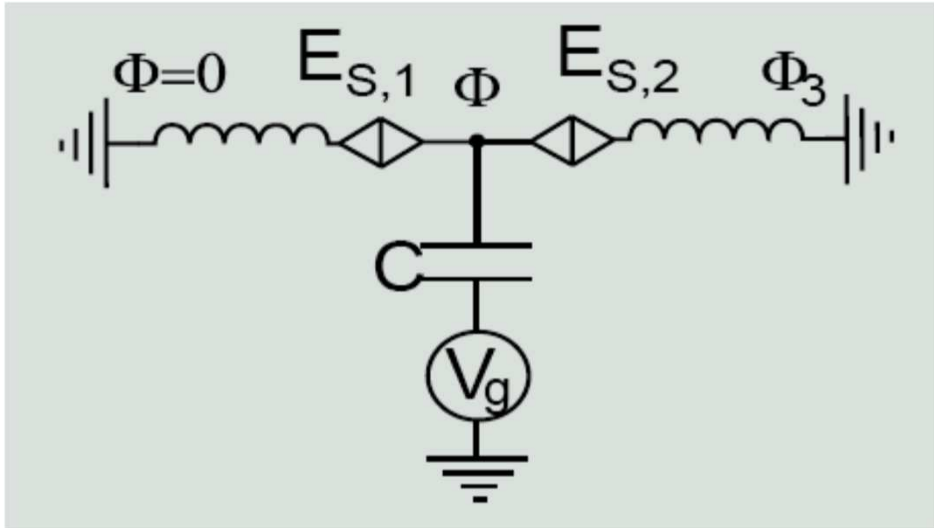
- Energies:
  - Inductive E_L
  - Charging E_C
  - PS E_s
  - ***More complex than box: topology***
- Hamiltonian
  - 2 charges:
  - unrestricted **Q**+periodic **q**

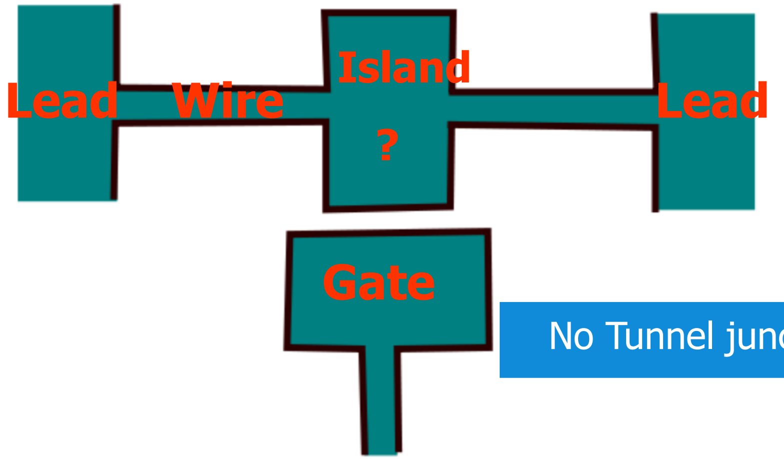
$$E_S^{(1)}\cos(2\pi(Q+q))+$$

$$E_S^{(2)}\cos(2\pi(Q-q))$$

$$E_C\left(Q-q_0\right)^2/2+$$

$$E_L\left(-\partial_Q^2-\left(\partial_q-i\phi_3\right)^2\right)/2$$

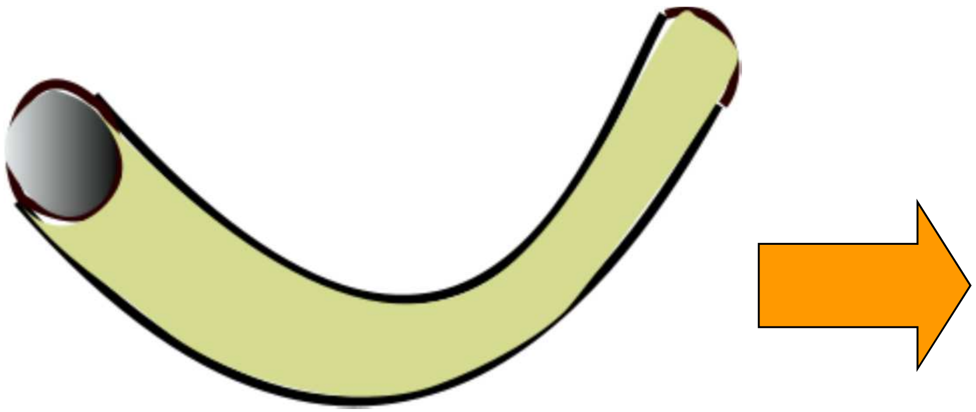


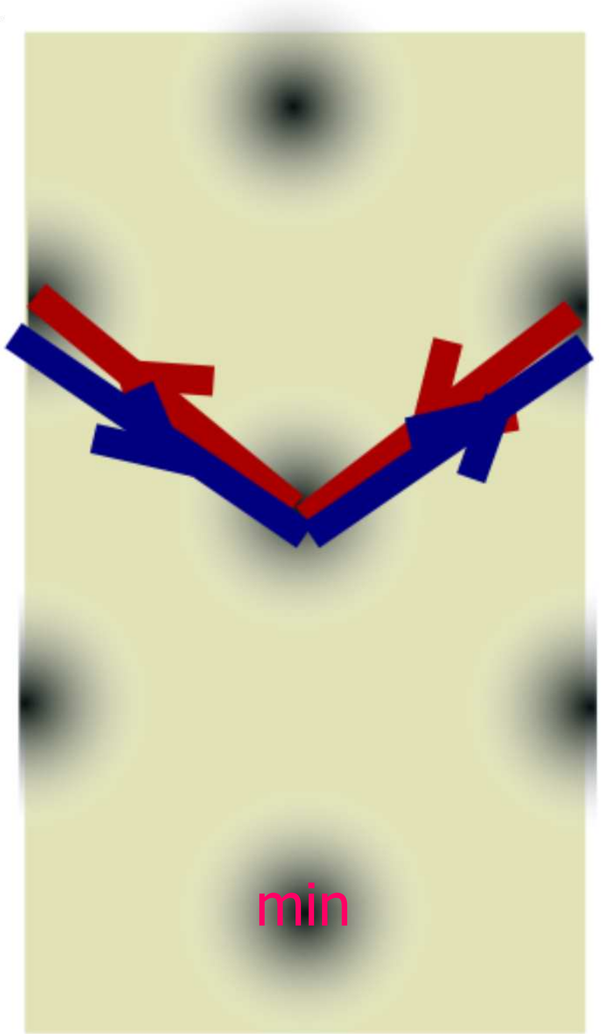


No Tunnel junctions

Limits

- Parabolically curved tube
- Small E_s *–small potential:*
  - oscillator+ offset
- Big E_s *–big potential:*
  - localisation, oscillators, tunneling, interference





Peculiarities

- From oscillator
  - to oscillators
- Charge sensitivity
  - second-order in E_s
  - Developed isolation
    - Exp-small supercurrent

How about duality?

- CPB is not dual to phase-slip CPB
- There are Josephson-based analogues with dual Hamiltonian
- Those are rather unnatural
  - Jens Koch et al. Phys. Rev. Lett. **103**, 217004 (2009)

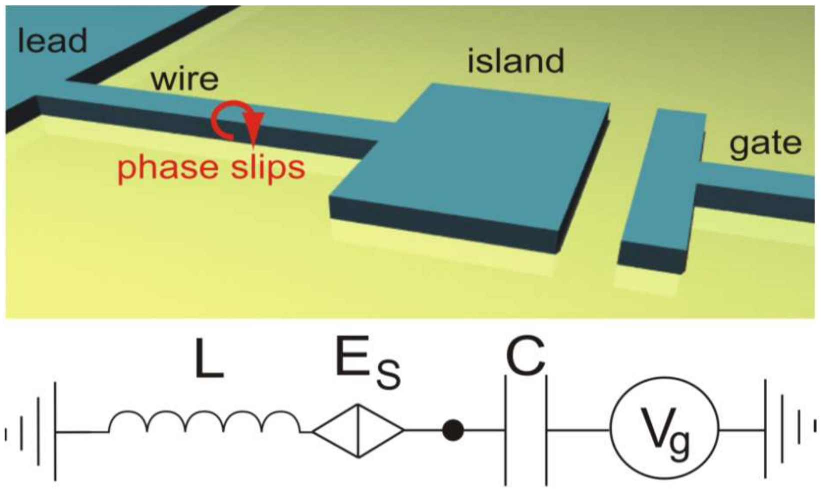
Hunting vanishing phase-slip amplitudes

- No reliable microscopic theory
- E_s unpredictable
- Big chance it's small
- For devices described, $E_s \sim E_L, E_C$
  - Bad in comparison with incoherent processes
- There is a trick!
  - Use of oscillators for sensitive measurements

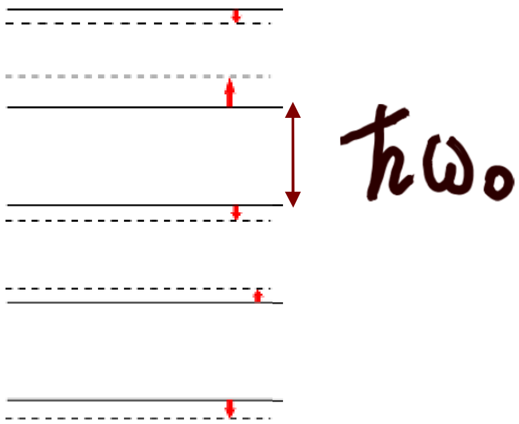
Phase-slip oscillator

Damped LC oscillator + phase-slips +a.c drive

Γ : Damping rate



$E_S << \omega_0$



Sensitivity:

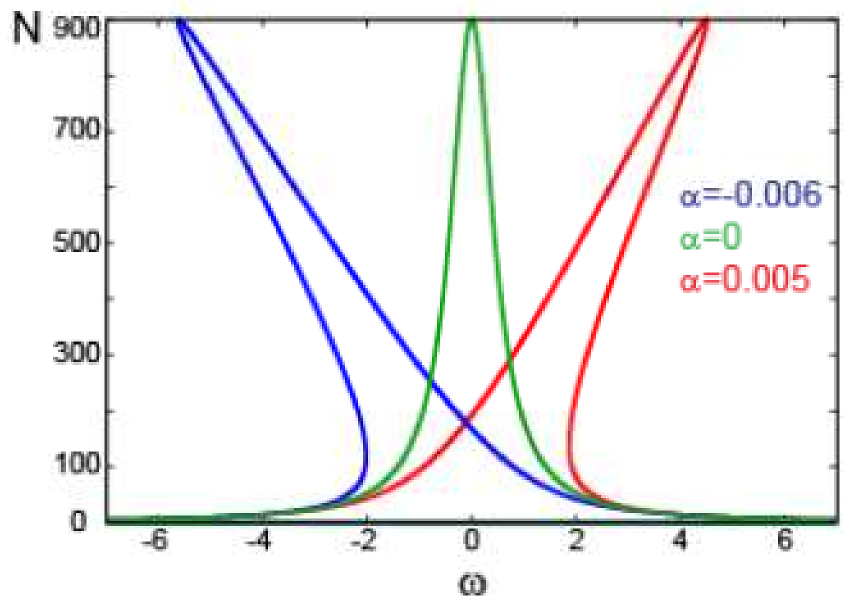
- ***up to Es*** $\sim \Gamma$

Non-linearities: Duffing oscillator

- Non-linearities:
  - Widespread
  - Applied
  - Simple functions

  of N, number of photons

  - Shift of res. frequency
    - $\omega \rightarrow \omega + \alpha N$
    - Causes bistabililty



Phase-slip oscillator: unusual non-linearities

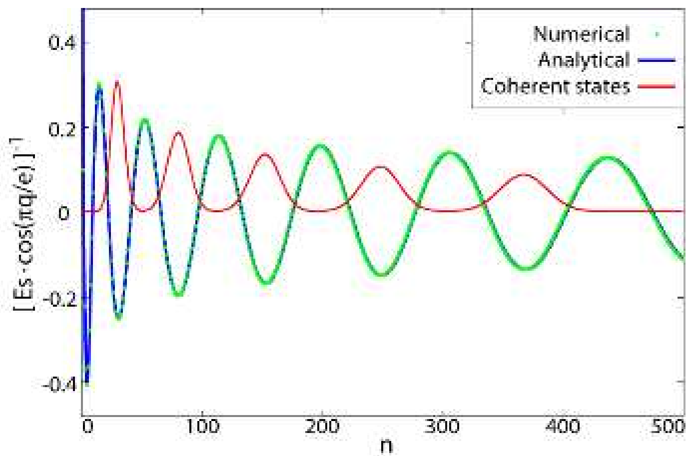
- Correction to the energy levels

$$\gamma = \sqrt{\frac{\hbar\pi^2}{2e^2}\sqrt{\frac{C}{L}}} = \sqrt{\frac{\pi}{2G_Q Z}} \gg 1$$

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right) + 2E_S \cos(\pi q/e)\frac{\cos(2\gamma\sqrt{n} - \frac{\pi}{4})}{\sqrt{\pi\gamma}\cdot n^{1/4}}$$

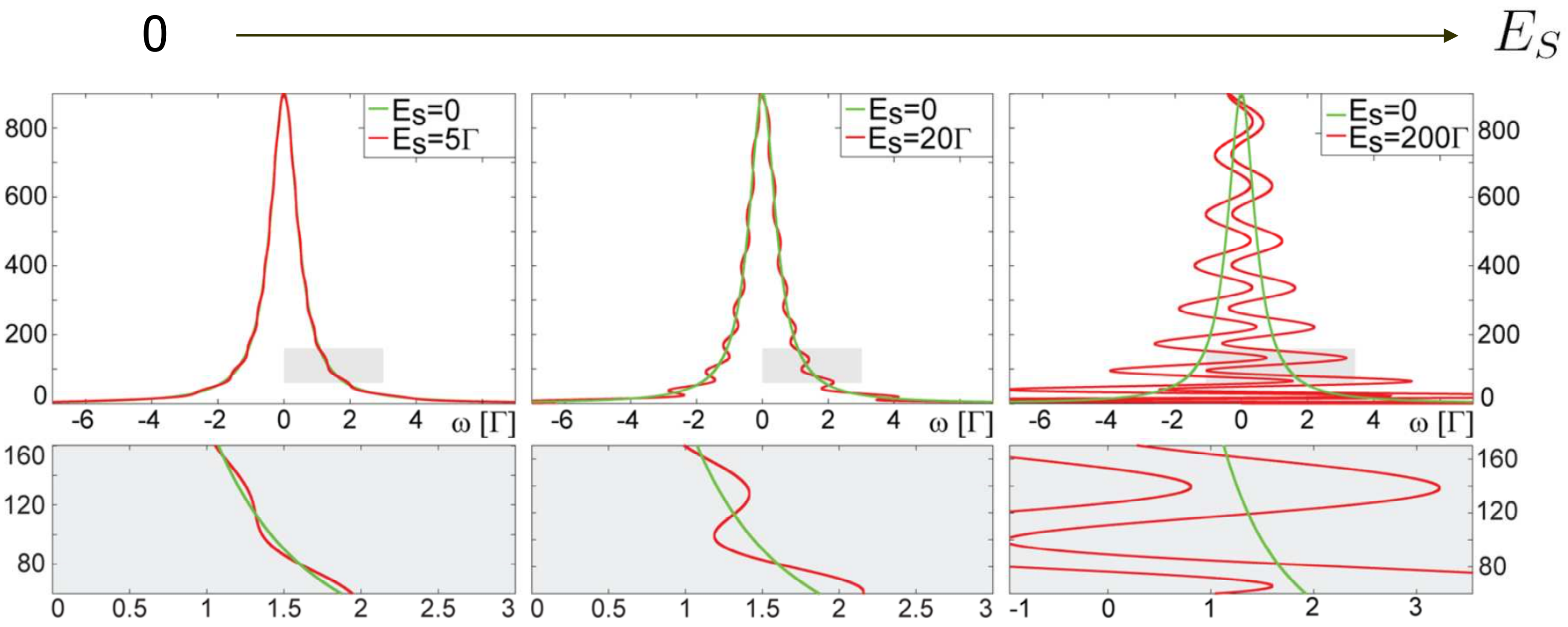
Charge sensitivity:
CB signature

Unusual non-linearities



- Makes it more interesting

Semiclassical realm: multiple stability



- Frequency shift oscillates with N

Quantum realm: questions

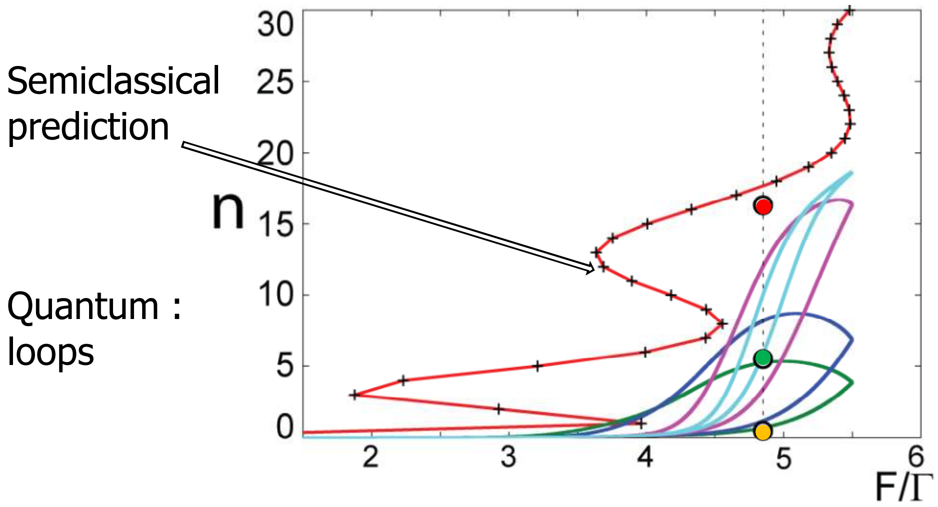
- Stable classical solution: how many quantum states are there?
  - Could be many: driven system
  - If few: can we manipulate?
- Answers in density matrix equation:

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{H}_R, \hat{\rho}] + \Gamma\left(b\hat{\rho}b^\dagger - \frac{1}{2}(b^\dagger b\hat{\rho} + \hat{\rho}b^\dagger b)\right)$$

$$\hat{H}_R = E(b^\dagger b) + \frac{\hbar F b^\dagger + \hbar F^* b}{2} + \hbar\omega b^\dagger b$$

  - numerics

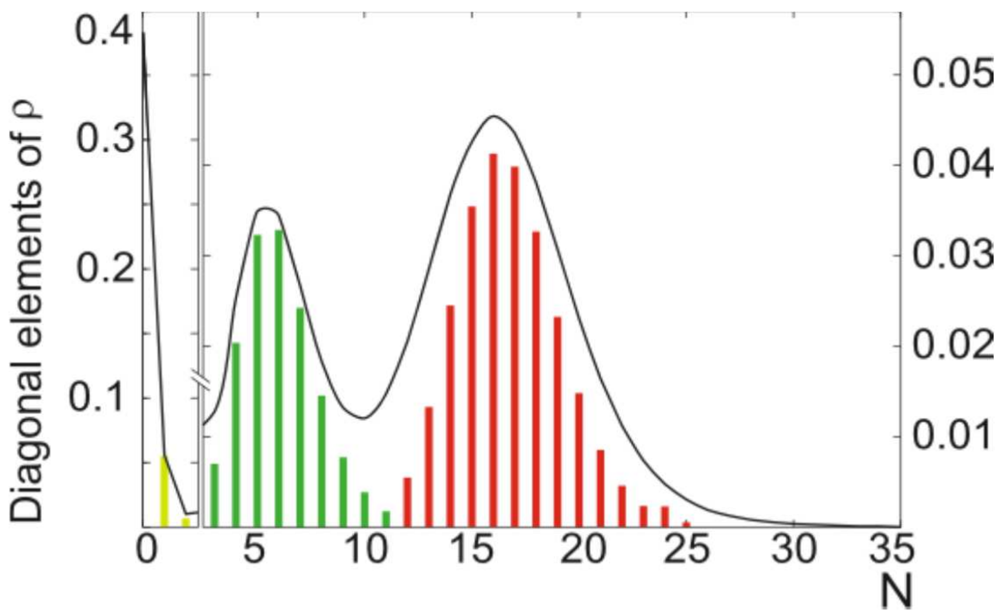
Hysteresis in quantum regime



- Expected equilibration time: $1/\Gamma$
- Sweep drive amplitude 0 to 5.5 back and forth
- Hysteresis at 10 000 $1/\Gamma$
- Interpretation: Exp. slow **switching** between classical stable solutions

Pure states versus stable points

- Diagonalize density matrix
- 90% in three states
- **Dark**, **coherent**, **coherent**
- One quantum state per classical solution
- Manipulation: remains to be investigated



Summary

- Fascinating isolation
- To be observed through making devices
- To look at phase-slip
  - CP box
  - CP transistor
  - Especially, oscillator